Bisimulation minimisation mostly speeds up probabilistic model checking

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University of Twente [1]
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March 28, 2007

Probabilistic model checking

1. Enjoys a rapid increase of interest
2. Case studies:
   - Biological process modeling
   - Communication protocols
   - Randomised algorithms
   - Quantum computing
   - Planning and AI
   - Security
3. Formalisms that use probabilistic model checking:
   - Probabilistic extension of Promela (Baier et al., 2005a)
   - Stochastic process algebra PEPA (Hillston, 1996)
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   - LiQuor (Baier et al., 2005a)
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Probabilistic model checking

1. State-space explosion
2. State-space reduction techniques:
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- Huge state-space reduction
- Is fully automated
- Drastic time penalty for LTL model checking (Fisler and Vardi, 1998; Fisler and Vardi, 1999; Fisler and Vardi, 2002)

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An empirical study

We did an empirical study on the effect of bisimulation minimization on probabilistic model checking.

Our main result

Bisimulation minimization often pays off.

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- Discrete and continuous time Markov Chains
- Reward extensions

An empirical study

- Use benchmark problems in the field (Kwiatkowska et al., 2007)
- Investigate 7 case studies
- Perform about 1870 experiments

Focus on

- The state-space reduction
- Time of lumping + verification
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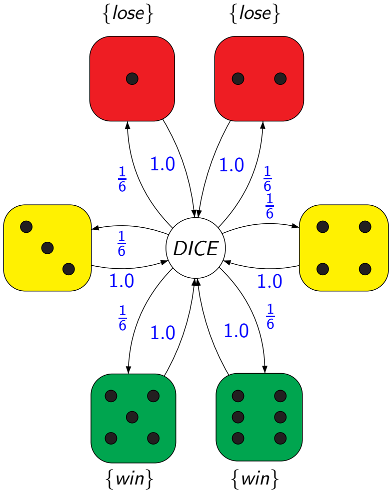
The considered models

Definition (Discrete time Markov chain)

A (labelled) DTMC is a tuple $(S, \mathcal{P}, AP, L)$:

- S - a finite set of *states*,
- AP - a finite set of *atomic propositions*,
- $L : S \rightarrow 2^{AP}$ - a *labelling* function,
- $\mathcal{P} : S \times S \rightarrow [0, 1]$ - a *probability matrix*,

$$\sum_{s' \in S} \mathcal{P}(s, s') = 1 \text{ for all } s \in S$$



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- Continuous time Markov chains
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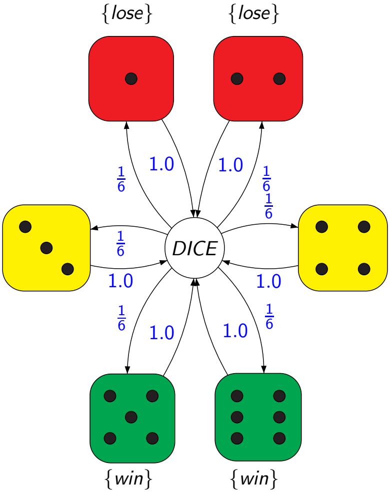

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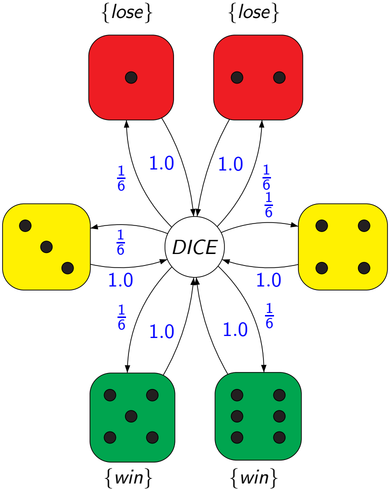
Probabilistic time-bounded reachability

Example

Determine states from which *win* states may be reached with a probability at least 0.9, within 10 time steps.

$$\mathcal{P}_{\geq 0.9}(\Diamond^{\leq 10} win)$$

Model	Example
DTMC	$\mathcal{P}_{\geq 0.9}(\Diamond^{\leq 10} win)$
CTMC	$\mathcal{P}_{\geq 0.9}(\Diamond^{\leq 3.5} win)$
Rewards	$\mathcal{P}_{\geq 0.9}(\Diamond^{\leq 15}_{\leq 13.7} win)$



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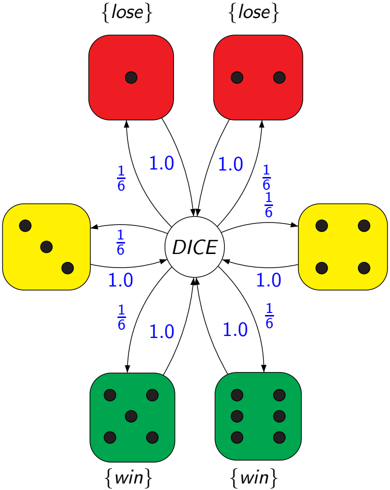

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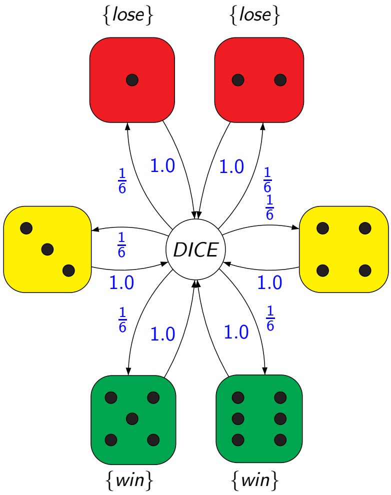

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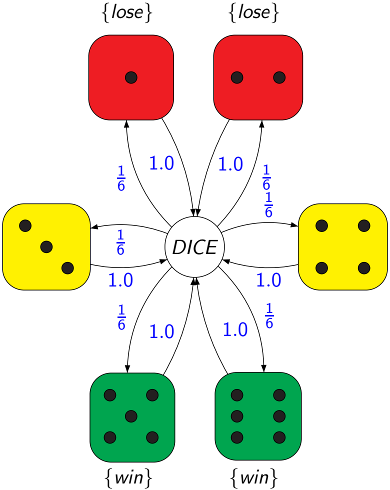

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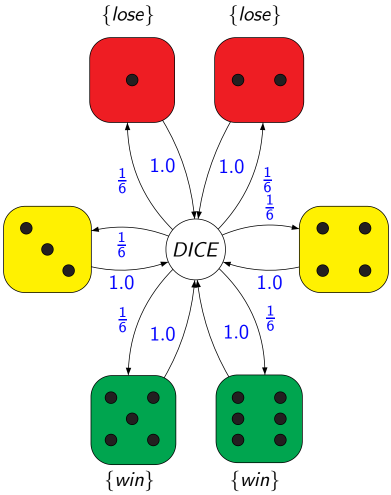

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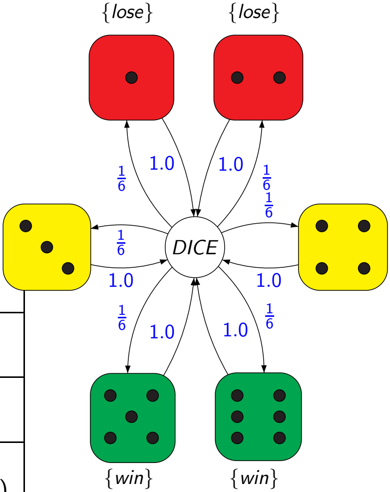
CTMC	CSL (Baier et al., 2003)
Rewards	PRCTL/CSRL (Andova et al., 2003; Baier et al., 2000)

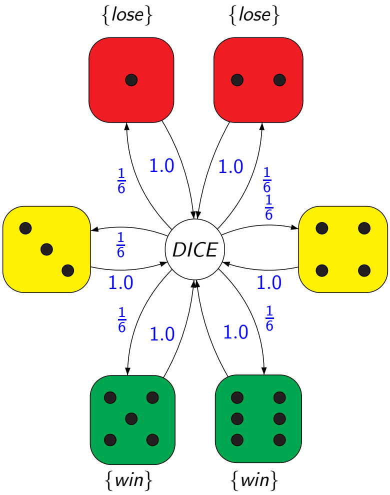


Bisimulation minimization

Definition (Strong bisimulation (Buchholz, 1994; Hillston, 1996))

- Let $D = (S, \mathcal{P}, AP, L)$ be a DTMC.
- Δ an equivalence relation on S.
- S/Δ is the *quotient* of S under Δ.
- Δ is a *strong bisimulation*, if $s_1 \Delta s_2 \Rightarrow$

$$L(s_1) = L(s_2)$$
$$\forall B \in S/\Delta \ : \ \mathcal{P}(s_1, B) = \mathcal{P}(s_2, B)$$



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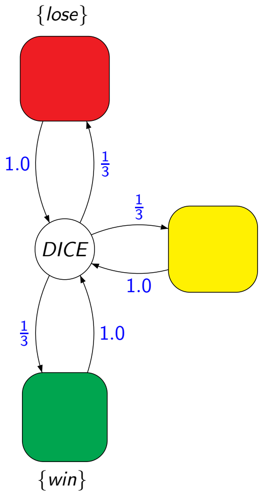

Preservation results

Theorem (1, (Aziz et al., 1995))

Let D be a DTMC, Δ a bisimulation and $s \in S$. Then $\forall \Phi \in PCTL^*$

$$s \models_D \Phi \iff [s]_\Delta \models_{D/\Delta} \Phi$$

Note

- Probabilistic bisimulation is the coarsest relation for Theor. 1.
- Since $s \sim [s]_\Delta$, verify properties on a bisimulation quotient.

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Definition (F-bisimulation (Baier et al., 2000))

- Let $D = (S, \mathcal{P}, AP, L)$ be a DTMC.
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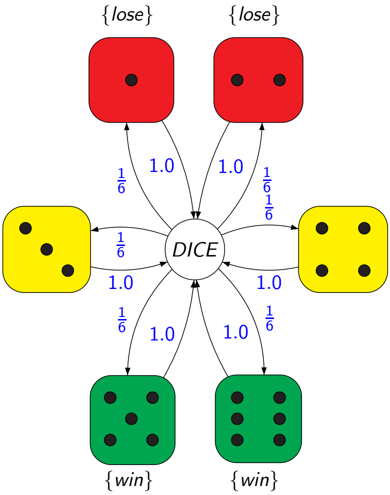
Let us take $F = \{win\}$.



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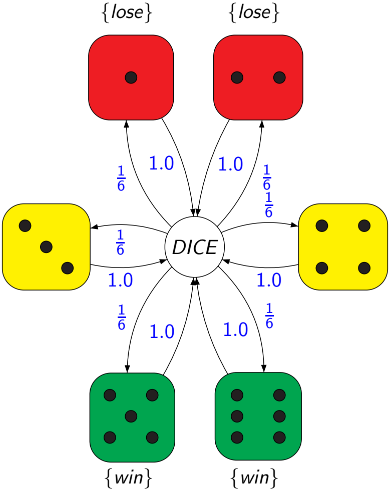

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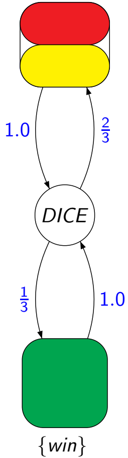

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- The worst-time complexity is $O(|P|log|S|)$

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Initial partitioning for $\mathcal{P}_{\trianglelefteq p}(\Phi \, \mathrm{U} \, \Psi)$ and $\mathcal{P}_{\trianglelefteq p}(\Phi \, \mathrm{U}^{[0,t]} \, \Psi)$

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- F – bisimulation: Formulas Φ, Ψ

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- Choose $F = \{U_0,\ S_1,\ S \setminus (U_0 \cup S_1)\}$.
- Apply F-bisimulation.

Initial partitioning for $\mathcal{P}_{\trianglelefteq p}(\Phi \mathrm{U} \Psi)$ and $\mathcal{P}_{\trianglelefteq p}(\Phi \mathrm{U}^{[0,t]} \Psi)$

Note

- Strong bisimulation: Atomic propositions
- F – bisimulation: Formulas Φ, Ψ

S_1 vs. U_1

A finer initial partitioning

$\mathcal{P}_{\trianglelefteq p}(\Phi \mathrm{U} \Psi)$

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- Choose $F = \{U_0, U_1, S \setminus (U_0 \cup U_1)\}$.
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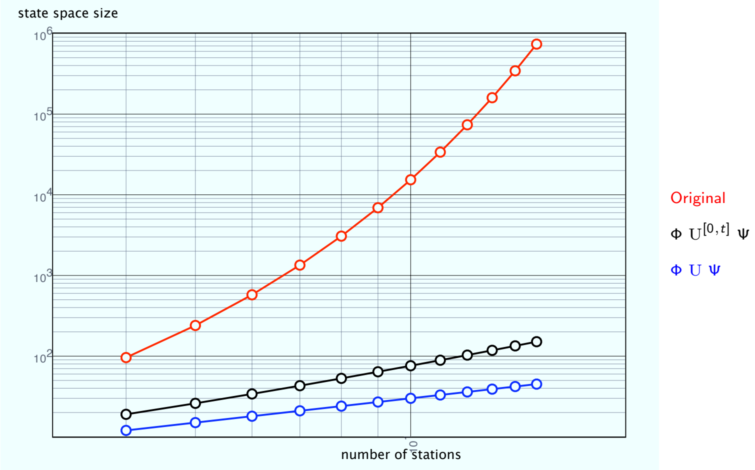
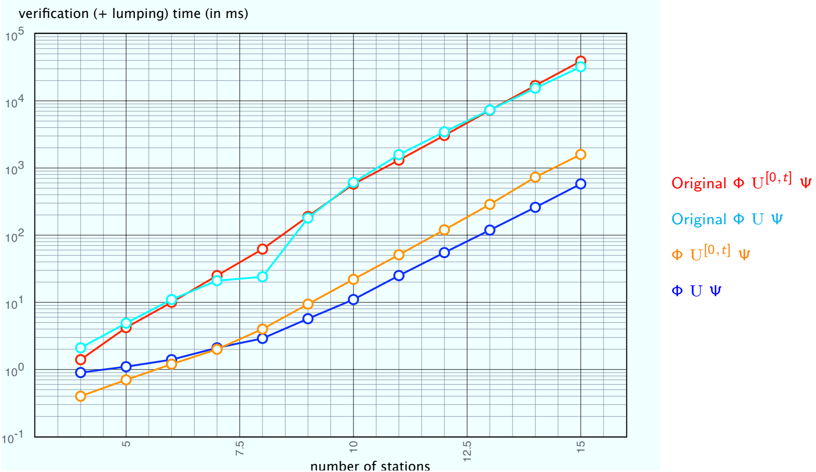
Cyclic polling server (Ibe and Trivedi, 1990)



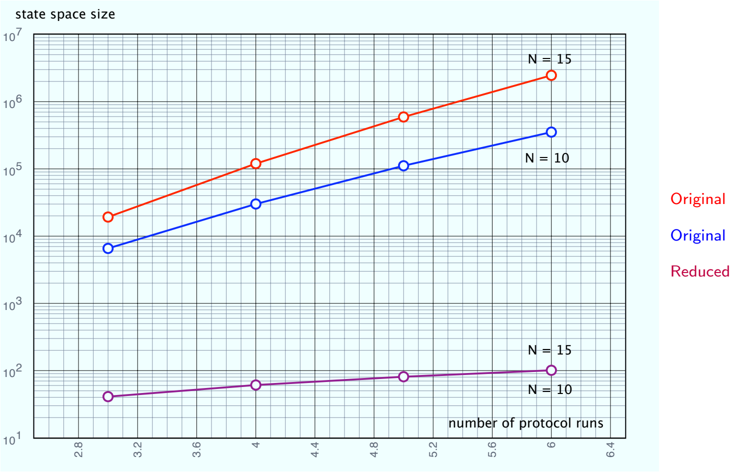
State-space reductions for $\mathcal{P}_{\leq q}(\neg serve_1 \ \mathrm{U}^{[0,1010]} \ serve_1)$ and $\mathcal{P}_{\leq q}(\neg serve_1 \ \mathrm{U} \ serve_1)$

Cyclic polling server (Ibe and Trivedi, 1990)



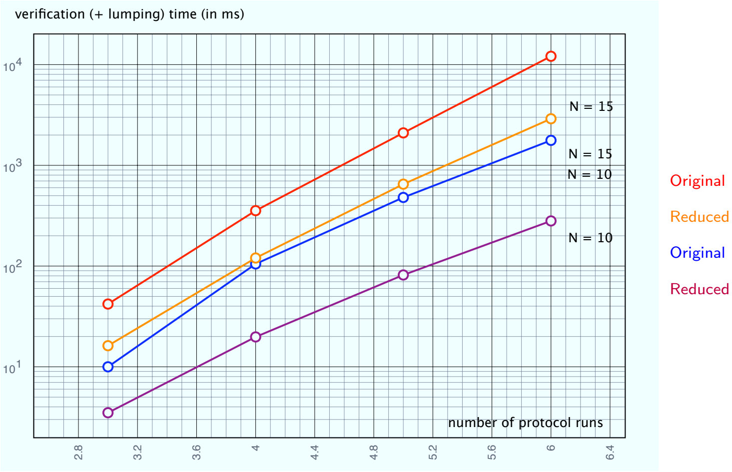
Run times for $\mathcal{P}_{\leq q}(\neg \mathit{serve}_1\ \mathrm{U}^{[0,1010]}\ \mathit{serve}_1)$ and $\mathcal{P}_{\leq q}(\neg \mathit{serve}_1\ \mathrm{U}\ \mathit{serve}_1)$

Crowds protocol (Reiter and Rubin, 1998)



State-space reductions for eventually observing the real sender more than once

Crowds protocol (Reiter and Rubin, 1998)



Run times for eventually observing the real sender more than once

Simple P2P protocol (Kwiatkowska et al., 2006)

			symmetry reduction (Kwiatkowska et al., 2006)				
original CTMC			reduced CTMC			red. factor	
N	states	ver. time	states	red. time	ver. time	states	time
2	1024	5.6	528	12	2.9	1.93	0.38
3	32768	410	5984	100	59	5.48	2.58
4	1048576	22000	52360	360	820	20.0	18.3

			bisimulation minimisation				
original CTMC			lumped CTMC			red. factor	
N	states	ver. time	blocks	lump time	ver. time	states	time
2	1024	5.6	56	1.4	0.3	18.3	3.3
3	32768	410	252	170	1.3	130	2.4
4	1048576	22000	792	10200	4.8	1324	2.2

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The end

Concluding remarks

- Significant, up to logarithmic, state-space reduction.
- The abstraction technique is fully automated.
- Strong bisimulation:
  - Sometimes, a substantial model-checking time reduction.
  - Sometimes, an increase of peak memory (by 50%).
- F-bisimulation:
  - Sometimes, a substantial model-checking time reduction.
  - The peak memory use is typically unchanged.
  - For reward case a decrease of peak memory (by 20-40%).

Future work

- Combine symmetry reduction with bisimulation.
- Extend experiments towards MDPs and simulation preorders.

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