A Markov Reward Model Checker

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September 20, 2005

Verification of Markov Reward Models

MRMs are underneath:

- Reward extensions of stochastic process algebras
- Stochastic reward nets
- Etc.

Rewards:

- State rewards
- Impulse rewards

  ▶ Example

Logics:

PRCTL - Probabilistic Reward Computation Tree Logic,
PCTL $\subset$ PRCTL.

CSRL - Continuous Stochastic Reward Logic,
CSL $\subset$ CSRL.

Allowed properties

PRCTL extends PCTL with:

1. The expected reward rate at a time instant

2. The long-run expected reward rate per time unit

3. The instantaneous reward at a time instant
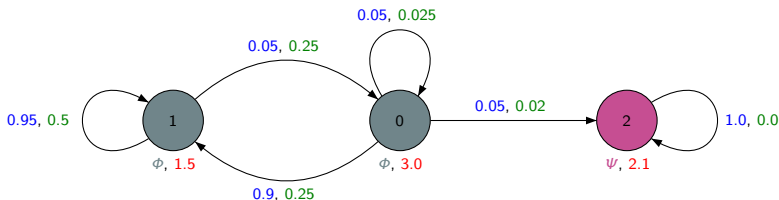
4. The expected accumulated reward at a time instant

CSRL extends CSL with

1. PRCTL

2. The probability to reach one of the goal states (via indicated allowed states) within time t while having earned an accumulated reward that does not exceed r is larger than p

Examples

Example: PRCTL

$\mathcal{Y}^3_{[2,5]} \Phi$ - the expected accumulated cost in Φ-states within 3 hops is between 2 and 5

Example: CSRL

$\mathcal{P}_{\geq 0.3}(\Phi \, \mathcal{U}^{\leq 4}_{(23,\infty)} \, \Psi)$ - a Ψ-state is reached with probability at least 0.3 in at most 4 time units along an Φ-path with total cost > 23

Implementation

Algorithms:

Sparse matrix - compressed row, compressed column
PCTL   - (Hansson and Jonsson, 1994)
PRCTL   - (Andova et al., 2003)
CSL   - (Baier et al., 2003)
CSRL   - Discretization (Tijms and Veldman, 2000),
              Uniformization (Qureshi and Sanders, 1996)

Improvements:

- Search for bottom strongly connected components
- On-the-fly steady state detection
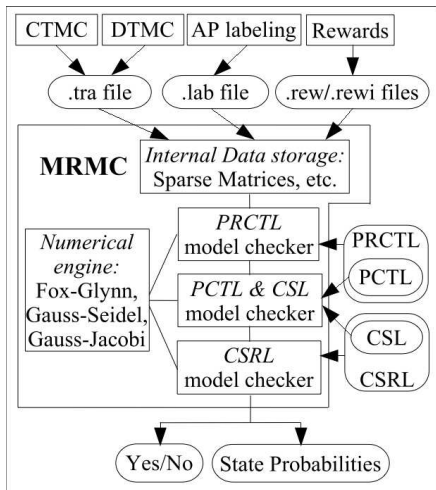- Path graph representation
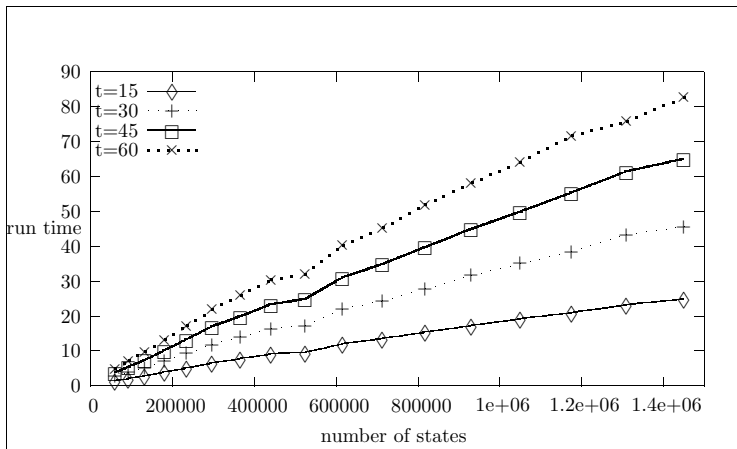
A command-line tool

Input files:

- `.tra` - the probability/rate matrix
- `.lab` - the state-labeling
- `.rew` - the state rewards
- `.rewi` - the impulse rewards

Example

```
46:)ewi410:/> ../../../bin/mrmc csrl
csrl_unif_state_impulse_01.tra csrl_unif_state_impulse_01.lab
csrl_unif_state_impulse_01.rew csrl_unif_state_impulse_01.rewi
Running in the CSRL model checking mode.
States=5, Transitions=11
Space Occupied:: 1424 Bytes.
Type 'help' to get help.
>>P[>0.1][ Sup U[0,50][0,3000] failed ]
State = 0, Probabilty = 5.087386e-03, Error Bound = 5.325909e-10
State = 1, Probabilty = 1.230380e-02, Error Bound = 4.466197e-10
State = 2, Probabilty = 1.000000e-00, Error Bound = 4.147793e-12
State = 3, Probabilty = 1.000000e-00, Error Bound = 4.147793e-12
State = 4, Probabilty = 1.000000e-00, Error Bound = 4.147793e-12
Result: ( 5.08738612501904e-03, 1.230379607517677e-02,
9.99999999958522e-01, 9.99999999958522e-01,
9.99999999958522e-01 )
time to compute: 953 micro sec(s)
states = {3, 4, 5, }
>>quit
```

CSL logic: $\Phi \ U^{[0,t]} \ \Psi$

CSRL logic: $\Phi \, U_{[0,r]}^{[0,t]} \, \Psi$

computation time (in s)

error bound: 10^{-3} ——
10^{-4} ··+··
10^{-5} ——

state space
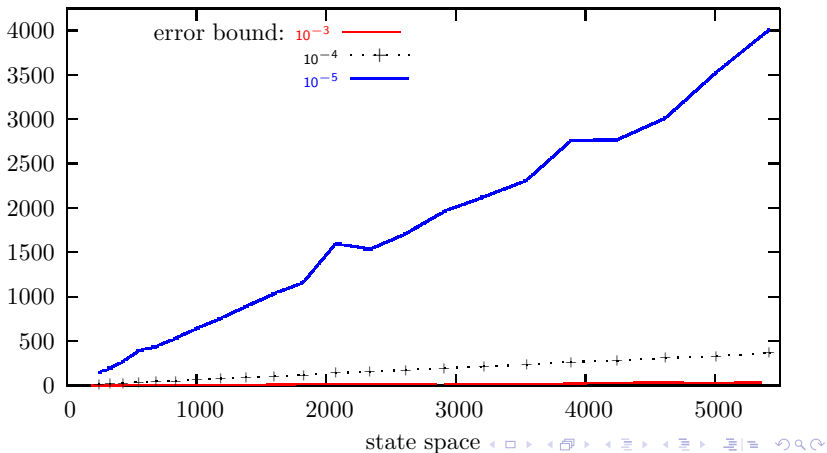
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Tijms, H. C. and Veldman, R.: 2000,
A fast algorithm for the transient reward distribution in continuous-time Markov chains,
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State and impulse rewards

A command-line tool

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